Upon Impact Numerical Modeling of Foam Materials

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The paper presents some theoretical and practical issues, particularly useful to users of numerical methods, especially finite element method for the behaviour modelling of the foam materials. Given the characteristics of specific behaviour of the foam materials, the requirement which has to be taken into consideration is the compression, inclusive impact with bodies more rigid then a foam material, when this is used alone or in combination with other materials in the form of composite laminated with various boundary conditions. The results and conclusions presented in this paper are the results of our investigations in the field and relates to the use of LS-Dyna program, but many observations, findings and conclusions, have a general character, valid for use of any numerical analysis by FEM programs.

Keywords: foam materials, rigidity, stress, strain, contact, material model

For their properties, foam materials are used more and more, even in those conditions where safety requirements must be full filled, in static and dynamic conditions. The foam materials category, also contains many materials with many differences between them, but with even more similarities.

The differences between the various foam materials, are mainly some physical characteristics and use. Many similarities come from their physical nature and especially from their technology and refers to the mechanical behaviours.

Such characteristics and using of foam materials in combination with other materials having very different characteristics (steel, aluminium, etc.) require special approaches in numerical analysis, from the choice of material model and continuing with all the details that need to be modelled in a numerical analysis .

This paper answers to such theoretical and practical issues.

Foam material characteristics

Foam materials represent a large material category, having a spongious and cellular structure. Such materials are sponge rubber, plastic foams, glass foams, refractory foams, and a few metal foams.

Urethane foam is a very known and used such material, which are produced from synthetic rubbers or plastics; urethane foams have 95% gas in closed microscopic pores. Other foam materials, like polyester foam, vinyl foams, silicone foam, epoxy foam, glass foam, ceramic foams, are also available for various industrial uses and types of materials.

We could say that any material that is manufactured by an expansion process is considered a foam and the base material is irrelevant.

Foam materials can be more or less - rigid, more or less - flexible, they have low weight, and are used in sheets with metal, paper, etc. in different domains. Such materials containing gas has low density and very low thermal conductivity.

One of the most important similarity between foam materials is the low density. This comes from expansion process of polymeric materials. Their main mechanical characteristic is the high compressibility expressed by the near zero value of the Poisson's ratio. This Poisson's ratio value makes the difference between foam and hyperelastic materials. These materials have large elongations but near no compressibility. The both materials are characterized by large deformations and large strains, so the approaching way of calculus is a nonlinear one. For a uniaxial stress state, the volume strain (e) is:

$$e \equiv \varepsilon_v = \frac{\Delta V}{V_0} = (1-2\nu)\varepsilon \quad (1)$$

It is clear that for a perfect incompressible material, $\Delta V = 0$, so $e=0$ but this is possible only for $\nu = 0.5$ (indeed, hyperelastic materials have Poisson's ratio near 0.50 value); for a perfect compressible material, $\Delta V = V_o$, so $e = 1$ but this is possible only for $\nu = 0$ (indeed, foam materials have Poisson's ratio near 0.0 value). Beyond these discrepancy between hyperelasticity and foam materials, something similar exist: the approaching way for the solving (stress and displacement calculus and others) of such problems. In the both cases, a strain energy function is used, and some specific strain measures are also used, next to the engineering strain ε. All these functions and parameters are expressed in terms of the principal stretches λ_1, λ_2, λ_3 which are defined like in the figure 1.

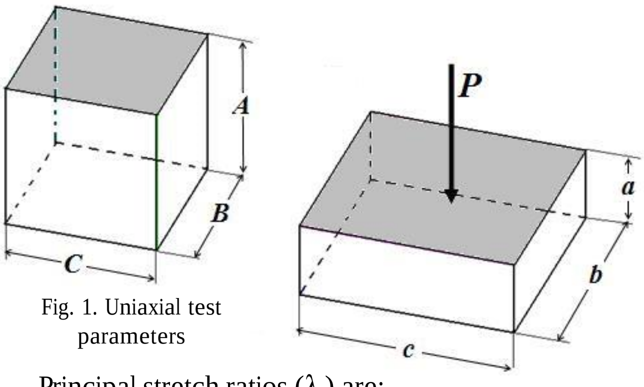

Fig. 1. Uniaxial test parameters

Principal stretch ratios (λ_i) are:

$$\lambda_1 = \frac{a}{A};\quad \lambda_2 = \frac{b}{B};\quad \lambda_3 = \frac{c}{C} \quad (2)$$

Both tension and compression, this parameter λ has only positive values. There is one difference: in compression, the parameter λ can not exceed and even touch the value of one, when in stretching, it can touch and even exceed the value of one. A tipical representation of the dependence between stress and stretch ration, in the case of compression (fig. 1) of a foam material is

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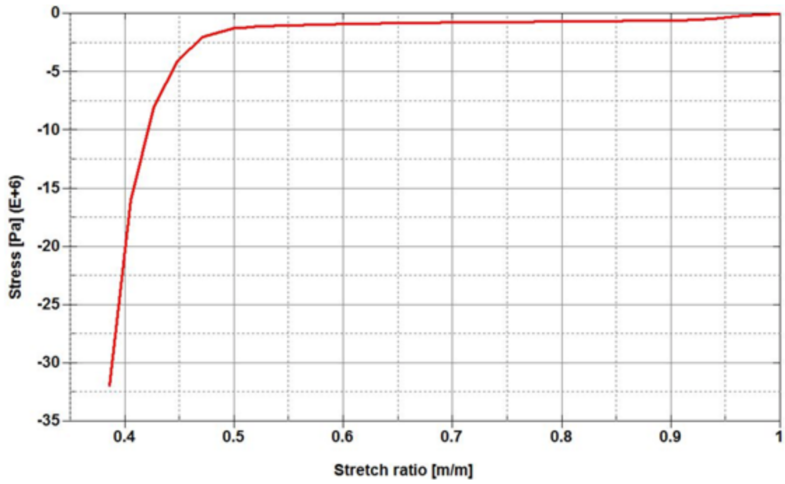

Fig. 2 Nominal stress-stretch ratio dependence for an uniaxial compression loading of a polymeric foam

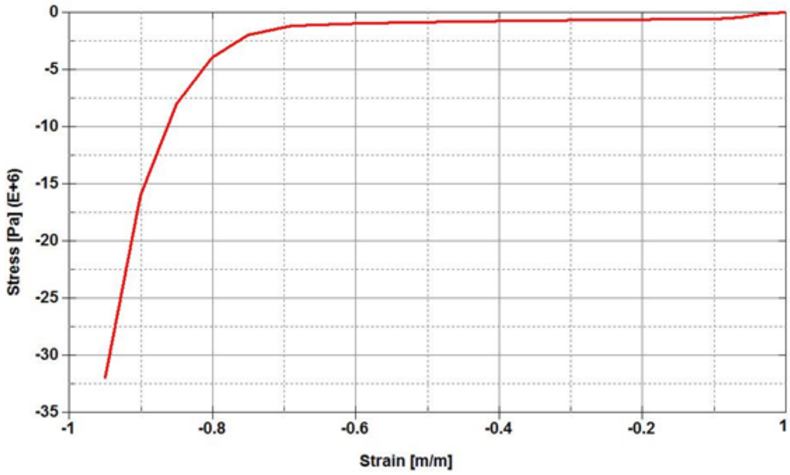

Fig. 3 Nominal stress-strain dependence for an uniaxial compression loading of a polymeric foam

presented in the figure 2, referring to the principal stretch ratio λ_1.

If the different behaviour of the foam material, in tension or in compression is neglected (an acceptable hypothesis for some foam materials and in some condition), the curve stress-strain (fig. 3) can be presented and discussed for positive domains of their values, like in the figures 5, 6 and 7.

For a square section, where B + C and b = c (fig. 1), $\lambda_2=\lambda_3$; also, for a circle section, where the characteristic dimension is the diameter (initial D and after loading d), $\lambda_2 = \lambda_3 = \frac{d}{D}$.

By an uniaxial test (fig. 1), the principal true stress σ can be written in terms of principal engineering stress (σ^E):

$$\sigma(\lambda) = \frac{P}{b \cdot c} = \frac{P}{\lambda_2 \lambda_3 \cdot B \cdot C} = \frac{P}{\lambda_2 \lambda_3 \cdot A_0} = \frac{\sigma^E}{\lambda_2 \lambda_3} \quad (3)$$

$$\sigma(\lambda) = \sigma^E \cdot \lambda_2^{-1} \cdot \lambda_3^{-1} \quad (4)$$

The strains can also be expressed in terms of principal stretches:

$$\lambda_1 = \frac{a}{A} = \frac{(1+\varepsilon_1)A}{A} = 1 + \varepsilon_1 \quad (5)$$

or,

$$\varepsilon_i = \lambda_i - 1 \quad (6)$$

This expression (6) is named in literature Biot strain or co-rotated engineering strain. In nonlinear analysis, with material nonlinearities and large deformations, others used strain measures are also used, as functions of principal stretch rations, like Green strain, Almansi strain and log (true) strain (relations (7) to (9), respectively).

$$\varepsilon_i = \frac{1}{2}\left(\lambda_i^2 - 1\right) \quad (7)$$

$$\varepsilon_i = \frac{1}{2}\left(1 - \frac{1}{\lambda_i^2}\right) \quad (8)$$

$$\varepsilon_i = \ln \lambda_i \quad (9)$$

The use of principal stretch rations (λ_1, λ_2, λ_3) is preferred because these are invariant with respect to both the coordinate system and the strain measure.

So, as it can be seen ε =f(λ) and ε has to increase monotonically with λ, for to be valid for mathematical operations (Taylor serie development etc.)

By above reasons, a strain energy function (φ) is also expressed in terms of λ_1, λ_2, λ_3. Such an energy function, $\varphi(\lambda)$, would have to full fill some conditions:

-to be zero for a ground state, when $\lambda_1 = \lambda_2 = \lambda_3 = 1$;

-to be symmetric in λ_i;

-to be always greater or equal then zero, $\lambda(\varphi) \geq 0$;

-to be a convex function.

A function, like $\varphi(\lambda)$, is called a convex function if any straight segment, joining two points, used in running analysis, is never below the graph or no intersection between curve and segment exist (fig. 4).

For stability, the energy function has to be a convex one, so any change in deformation field will produce an unique change in the stress field.

Defining the energy function $\varphi(\lambda)=\phi(\lambda_1, \lambda_2, \lambda_3)$ the variation of this function, with respect to λ can be written:

$$d\varphi(\lambda) = \frac{\partial \varphi}{\partial \lambda_1} d\lambda_1 + \frac{\partial \varphi}{\partial \lambda_2} d\lambda_2 + \frac{\partial \varphi}{\partial \lambda_3} d\lambda_3 \quad (10)$$

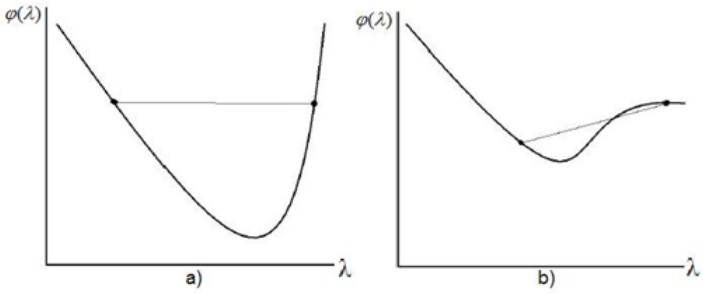

Fig. 4 Graphical representation of a convex (a) and non-convex function (b)

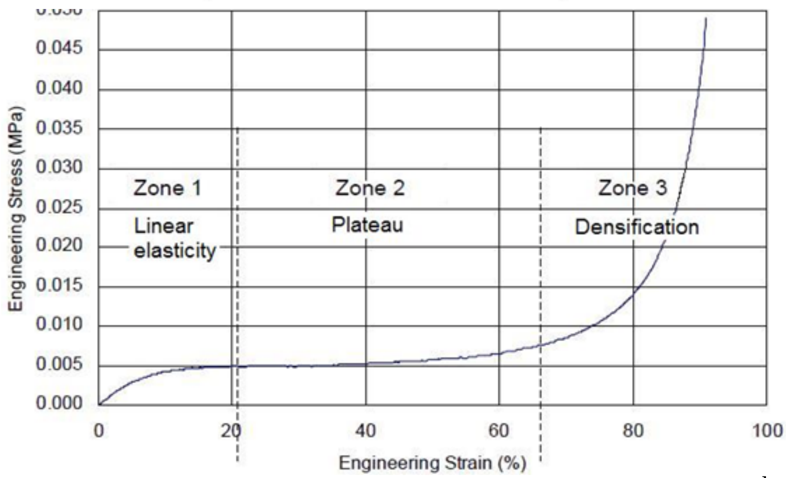

Fig. 5 A typical strain-stress curve of a material foam

The specific behaviour of the foam material is reflected by the stress-strain relationship. Such a typical curve is presented in the figures 3 and 5.

Three zones are noticed in the compressive stress-strain relationship of a foam material (fig. 5): an initial region (Zone 1), a flat plateau (like an yielding zone) compaction region (Zone 2) and the densification zone (Zone 3).

In the first zone, foams have some stiffness due to the strength of the matrix material itself. The curve is fairly linear and the stretch ratio of λ has the value range from 0.95 to 1. This zone is called the initial stiffness of the foam.

The yielding plateau (Zone 2) is the result of the gaseous component in foam structure. The gas exits the foam through the open pores or channels. In closed cell foams, the gas is compressed.

The stresses remain at about the same level, until λ reaches a value of 0.4-0.5, when by bending the cell walls collapse and a new zone appears. The Zone 3 (densification phenomenon) begins when the gas pressure is high enough to rupture the cell wall thereby releasing the gas to the atmosphere.

The damages of the cell walls are permanently in the foam material. The densification phenomenon makes the stress to rise steeply. Of course, if the matrix is strong enough, the cell remains intact but they collapse completely. In this zone, the foam begins to behave like the material in its stress-strain relationship.

Practically, it is not possible to get a stress-strain curve like in the figure 5 because not-passing difficulties could appear in experimental researching. By this reason, the experimental data are got only for Zone 1 and a part of zone 2; then, the curve is continued as the figure 6 shows.

It is very important that the stress-strain curve to cover the strain range occurring in reality; otherwise some *tricks* or better said some ways exist and these will be presented below in this paper.

The knowing the curve σ-ε, the curve slope, can be determined (evaluated) so the figure 7 shows (for first two zones).

Because of material nonlinearities and because of large deformations, nodal stresses and displacements are very difficult to be calculated by *classical* way of the finite element analysis. By this reason, such parameters are calculated starting from the energy function, φ(λ)and using some stress measures, next to Cauchy stress tensor; so, first and second Piola-Kirchhoff stress tensor, Jaumann stress tensor and others tensors or connection matrixes are used.

Each stress type is defined in connection with a strain type. For example, the second Piola-Kirchhoff stresses

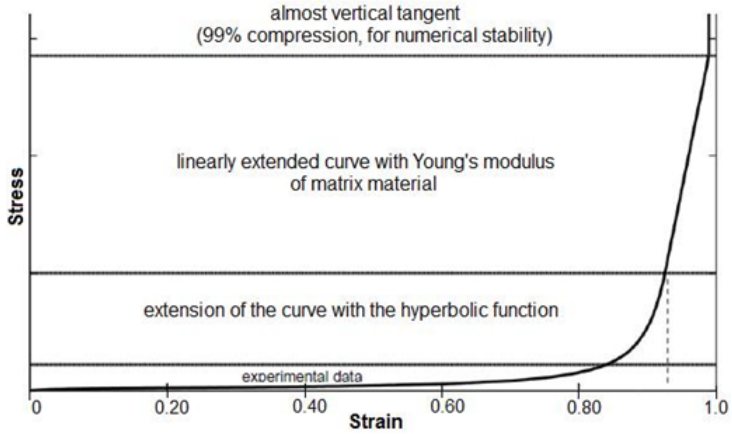

Fig. 6 The approaching ways for extension of the curve

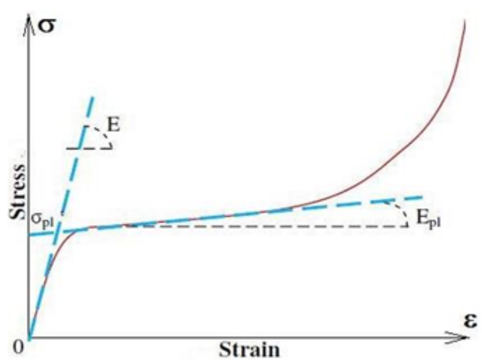

Fig. 7 Determination of Young'moduls

work with Green's strain etc. The engineering stress (σ^E - σ) can be written in the Hooke's law form:

$$\sigma_i^E = E \cdot \varepsilon_i \tag{10}$$

where E is the Young modulus. This relation (10) has to be understood in all its complexity, like a matrix form and for different stress states.

Like many others materials, the foam materials are also sensitive to the strain rate $\left(\dot{\varepsilon} = \frac{d\varepsilon}{dt}\right)$. By an uniaxial compressive test, at different strain rates, the stress-stretch ratio curves have allures presented in the figure 8.

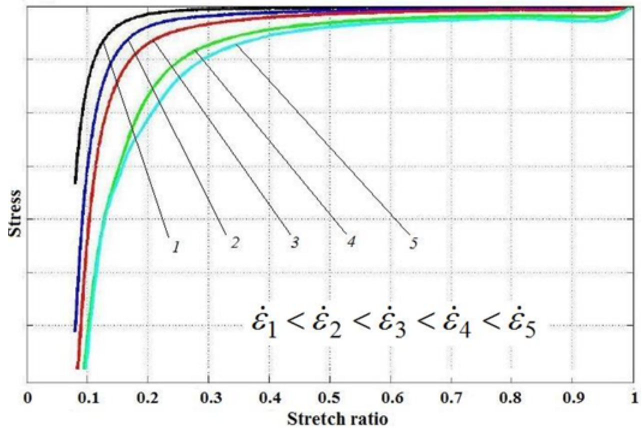

Fig. 8 The influence of the strain rate upon stress-stretch ratio

As we can see watching the figure 8, for the same value of the stretch ratio, the stress values increase with the strain rate. The quantitative values depend on the foam material and they must be experimentally determined.

Material models

Many professional programs, which offer the analysis possibility by FEM of the foam materials provides special material models. These material models have the same fundamentals which, in a synthetic way, will be presented below.

For all foam material models, the main theoretical issue is the energy functional $\varphi(\lambda)$. So, Hill's energy functional (one of the most used) is:

$$\varphi(\lambda) = \sum_{m=1}^{k} \frac{\mu_m}{\alpha_m}\left(\lambda_1^{\alpha_m} + \lambda_2^{\alpha_m} + \lambda_3^{\alpha_m} - 3 + \frac{1}{n}\left(J^{-\alpha_m n} - 1\right)\right) \tag{11}$$

where k is the number of terms in the function, n, α_m and μ_m are material constants, λ_1, λ_2, λ_3 are the principal stretches or the stretch ratio in the corresponding principal direction 1, 2, 3 and J is the relative volume (fig. 1):

$$J = \frac{V}{V_0} = \frac{a \cdot b \cdot c}{A \cdot B \cdot C} = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \tag{12}$$

For n=0, relation (11) becomes:

$$\varphi(\lambda) = \sum_{m=1}^{k} \frac{\mu_m}{\alpha_m}\left(\lambda_1^{\alpha_m} + \lambda_2^{\alpha_m} + \lambda_3^{\alpha_m} - 3\right) \tag{13}$$

representing the energy functional used in Ogden material model, often used in modelling of the hyperelasticity materials.

A very important observation was made by Storakers [4], according to the Hill's energy functional can be used to describe the foam material behaviour, where principal engineering stresses are uncoupled, depending only on the stretch ratio in the corresponding principal direction.

The principal Kirchhoff stress components can be determined [2] by the following relation coming from relation (11):

$$\tau_{ii}^K = \lambda_i \frac{\partial \varphi}{\partial \lambda_i} = \sum_{m=1}^{k} \frac{\mu_m}{J}\left(\lambda_i^{\alpha_m} - J^{-n\alpha_m}\right) \tag{14}$$

The Cauchy stress components (σ_{ij}) are then obtained:

$$\sigma_{ij} = J^{-1} \cdot \tau_{ij} \tag{15}$$

where τ_{ij} are the components of the standard Kirchhoff stresses obtained by the formula [2]:

$$\tau_{ij} = q_{ik} \cdot q_{jl} \cdot \tau_{kl}^K \tag{16}$$

The components q_{ij} are the elements of the orthogonal tensor containing the eigenvectors of the principal basis (coordinates corresponding with the principal stresses). Many other details are given in documentation of the used program.

In the material library of the Ls-Dyna program, some material models for foam materials exist: MAT_057 (Low Density Urethane Foam), MAT_083 (MAT_FU_CHANG_FOAM), MAT_177 (MAT_HILL_FOAM), MAT_178 (MAT_VISCOELASTIC_HILL_FOAM) and MAT_181 (Simplified Rubber/Foam). A special attention has to be paid to the selecting of the material model; the material properties have to be known but in the same time the goal of using of foam material has to be analysed.

Illustrative Examples

The illustrative examples presented below are going to answer at some questions which arise for any researcher interested in modelling by FEM of the impact with foam materials. Of course, each answer is closed related with the used program, but some recommendations or conclusions have a general validity.

A first question is referring to the type of finite element model. Mainly, there are three FE model types: 3D, 2D and

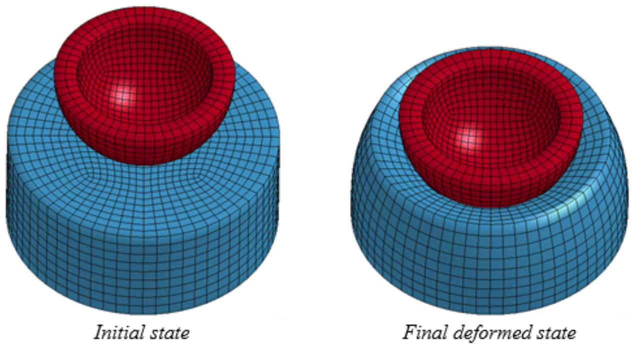

Fig. 9 The 3D finite element model

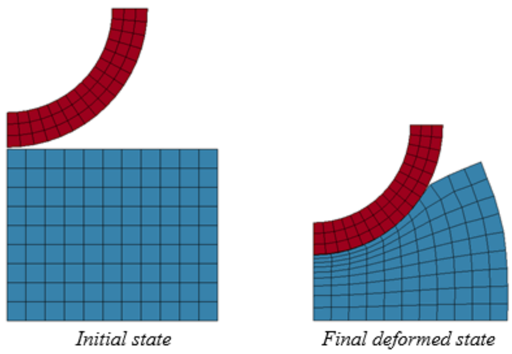

Fig. 10 The 2D axis-symmetric finite element model

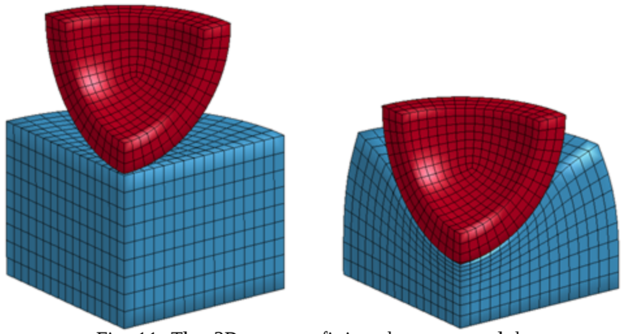

Fig. 11. The 3D quarter finite element model

reduced 3D models by taking into account the symmetry, if this exists. As the 2D models are concerned, these can be fitted for plane stress states, plane strain states and for axis-symmetric structures.

The results, presented bellow, are based on using of the Ls-Dyna program. The impact of a sphere (projectile) with a foam circular plate (target) is presented (figs. 9, 10 and 11), using some facilities offered by this program.

Figures 9, 10 and 11 present those three types of the finite element (FE) models, used in our researching (full 3D, 2D axis-symmetric and quarter 3D, respectively). The figures show both initial state of the models and final deformed state. The sphere has a diameter of 40 mm and a hypothetical mass of 5 kg. The plate diameter is 60 mm and the plate thickness is 25 mm. A low velocity impact is considered (5m/s). The 3D models used SOLID type elements with 8 nodes and the 2D model used PLANE162 finite element, with options of axisymetric and Lagrangian formulation.

The material model, used for modelling of the sphere, was a rigid material with the properties like a steel. The mass of 5 kg was appointed to the sphere by a special procedure of the program (mass trimming), not by changing the material density, a way with some negative influence upon contact. As it is seen in the figures 9 and 11, the sphere was modelled like a hollow sphere, in a half shape (fig. 9) or in a quarter of the half shapes (fig. 11).

For modelling of the foam plate, the low density foam material model was used was used, having the density of 10 kg/m^3 and Young modulus of $5 . 10^7$ Pa. The curve stress-strain of the foam material is presented in the figures 12 and 13. Using of the curve presented in the figure 12 – an initial curve – was not fitted, because just from the beginning an error appeared.

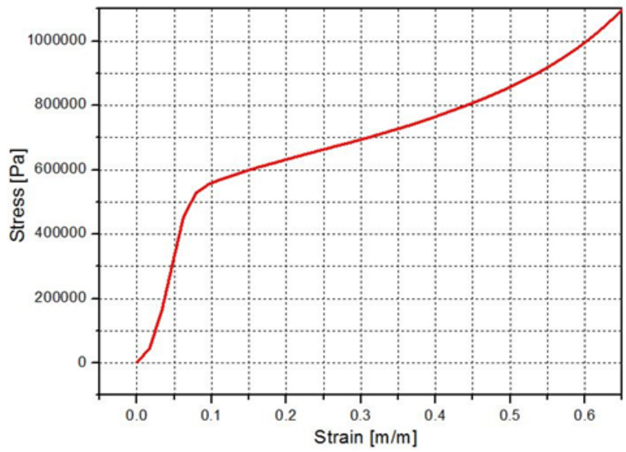

Fig. 12. The initial stress-strain curve of the foam material

Only one general constrain was applied: all the nodes of the bottom surface of the foam are fixed (no displacements, no rotations). For 2D and 3D_1/4 models, the specific constrains were applied, according to the specific of the model.

The results obtained using those three models presented above (fig. 9-11) are synthetically presented in the table 1.

The errors presented in the table 1 are referring to the comparing with the values obtained by using the full 3D model.

As the figure 13 shows, the sphere displacements (foam compression) obtained using those three FE models are very closed, practically between the results obtained by 3D and 3D_1/4 no difference exist.

The table 1 presents the errors only as the maximum values are concerned, and a very good concordance is noticed. The figures 13 and 15 show the concordance between parameters along the analysis time.

A very good concordance of the results is also noticed in figures 13 and 15. A conclusion can be formulated: any model presented here is available.

An other question, which appears for a researcher in analysis by FEM of an impact with foam materials, is how to pass over an error which stops the running with the message of *negative volume* and *Complex sound speed* and what has to be done for a right simulation of the contact.

For answering to these aspects, eight running versions were organised so these can be seen in the table 2. The full

Table 1
RESULTS OBTAINED BY THREE FINITE ELEMENT MODELS

Parameter	Measure unit	Finite Element Model Type		
		3D	2D	3D-1/4*)
Sphere Mass	kg	**4.99958**	4.99899	1.250
			-0.012 %	***0.008 %***
Foam Mass	kg	**0.00070575**	0.00069979	0.000174666
			-0.844 %	***-1.004 %***
UY Sphere Displacement	m	**0.014649**	0.014946	0.01463
			2.027 %	***-0.130 %***
Total Energy of the Foam	Nm	**9.1691**	$1.484*2*\pi$	2.347
			1.692 %	***2.387 %***
Von Mises Stress in the Foam	Pa	**$9.416*10^6$**	$9.564*10^6$	$9.80*10^6$
			1.572 %	***4.078 %***

*) The model 3D-1/4 is a quarter of the full 3D model (fig. 11).

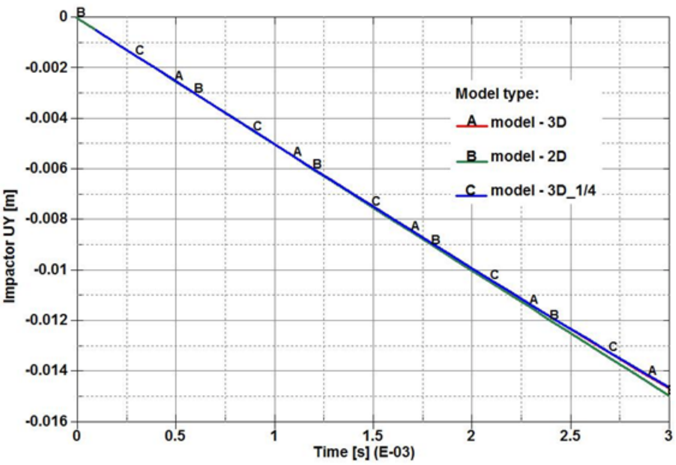

Fig. 13. Sphere displacements graphically compared

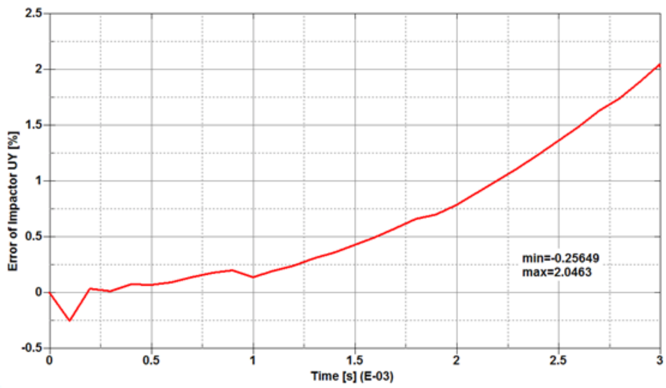

Fig. 14. Error time evolution between sphere displacements obtained by 3D and 2D models

Table 2
COMPARATIVE RESULTS FOR EIGHT RUNNING VERSIONS

No.	Initial curve	Expanded curve	SOFT 0	SOFT 1	KCON 0	KCON 2000	HGID 0	HGID 1	Effects
1	YES	---	YES	---	YES	---	YES	---	O.K.
2	YES	---	YES	---	YES	---	---	YES	NO_2
3	YES	---	---	YES	---	YES	YES	---	O.K.
4	YES	---	---	YES	---	YES	---	YES	O.K.
5	---	YES	YES	---	YES	---	YES	---	O.K.
6	---	YES	YES	---	YES	---	---	YES	O.K.
7	---	YES	---	YES	---	YES	YES	---	O.K.
8	---	YES	---	YES	---	YES	---	YES	O.K.

OK_1: Normal termination but the ***contact does not work*** beginning at the time t = 0.0028; UY=0.014624 m;

NO_2: Error, “negative volume” & “complex sound speed” at the time t = 2e-3;

NO_3: Normal termination but the ***contact does not work*** (UY=0.014987 m);

NO_4: Normal termination but the ***contact does not work*** (UY=0.014986 m);

OK_5: Normal termination and UY_5: 0.014617 m;

OK_6: Normal termination and UY_6: 0.014649 m;

OK_7: Normal termination but the ***contact does not work*** (UY=0.014986 m);

OK_8: Normal termination but the ***contact does not work*** (UY=0.014986 m).

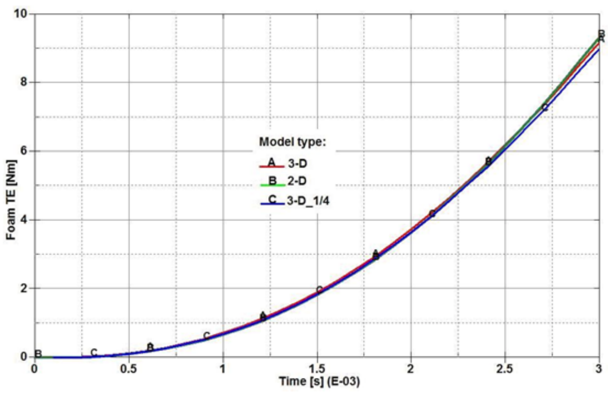

Fig. 15. Foam total energy graphically compared

3D model was used (fig. 9) with the same characteristics presented above.

Initial curve is presented in the figure 12. Expanded curve is presented in the figure 16. SOFT parameter is an optional component of the contact definition (contact automatic surface to surface). KCON is a parameter of the material definition of the foam material model (material low density foam).

HGID is a parameter of the part definition for the part representing the foam plate. More details about these parameters (soft, kcon, hgid) are presented in [6, 10].

For the calculus of the contact rigidity and the time step, LS-DYNA uses the maximum value of Young’s modulus, chosen between E_{foam} (given in material properties) and that value (E_{curve}) coming from the material curve. So,

$$E = \max|E_{foam}, E_{curve}|$$

This choosing of *E* ensures the stability of the computed time step, but the adopted value can be too small ($E > E_{curve}$) or too large ($E < E_{foam}$), so the computer time can be significantly influenced.

As the contact rigidity is concerned, looking at the figure 17, we can see that the main problem seems to be the contact rigidity.

The Young's modulus of the foam (E_{fqam}) has the value of $5.0*10^7$ [Pa]; analysing the stress-strain curve (fig. 12), permanently $E_{curve} < E_{foam}$, so $E = E_{foam} = 5 . 10^7$ [Pa] was used for the calculus of the contact rigidity and the time step.

The figure 16 together with the table 2 shows some very important aspects referring to the results obtained by eight adopted options for running of our problem. In the running of the versions No_3, No_4, No_7 and No_8 the contact does not work, not at all.

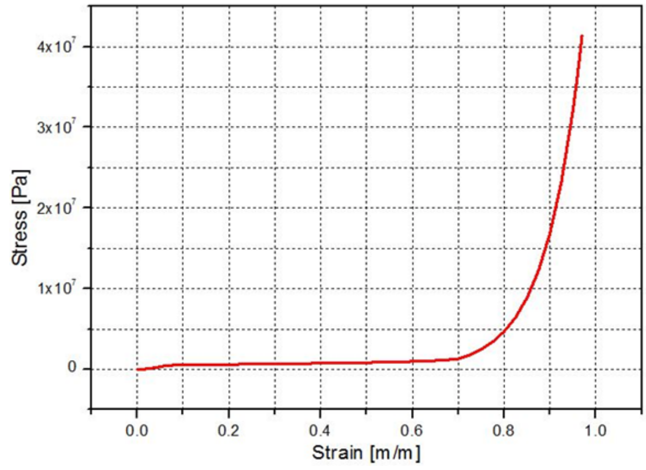

Fig. 16 Expanded stress-strain curve

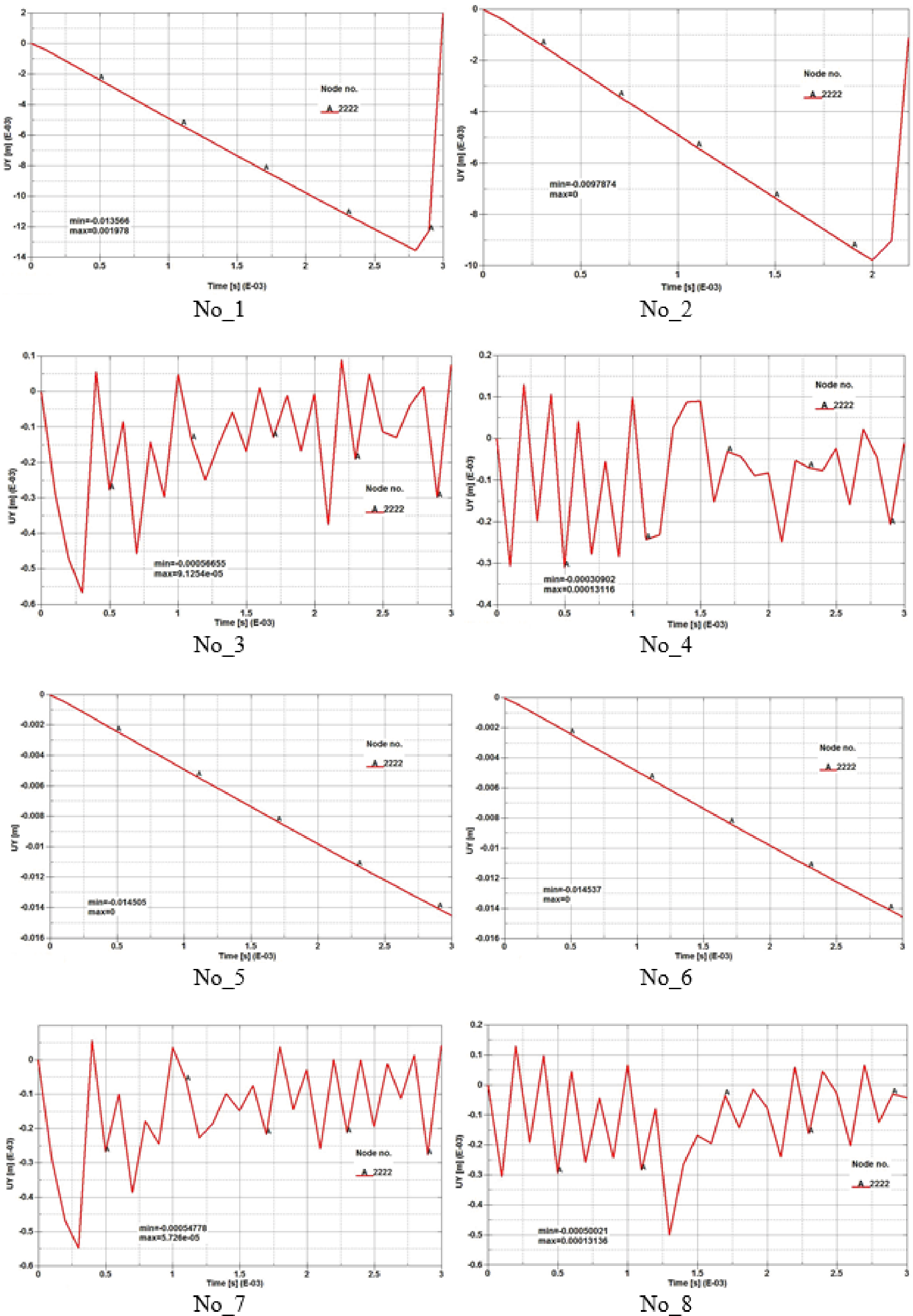

Fig. 17 The UY displacements of the foam central (impact) node

In the running of the version No_1, the contact simulation does not working only beginning at a time (t = 0.0028), before the final (analysis) time. In the running of the version No_2, at a time, the error *negative volume* and *complex sound speed* and the program stopped.

The first two running versions (No_1 and No_2) would have been OK if the analysis time would have been shorter. The running versions No_5 and No_6 are the best. Between them an error of 0.22 % exist, owing to the hourglass energy; the influence of this energy kind is insignificant.

In our researching, the using of different values of the parameter KCON did not lead to a right working of the contact, but lead to the passing over the error negative volume and complex sound speed.

Conclusions

The foam materials represent an especially type of materials having an increasing use in many industries. Often, such materials are subjected at different loads; the impact, which involves contact modelling, is a dynamic loading for which the strain rate can have an important influence (fig. 8). In this work, the impact velocity is at a low level (5 m/s), so the influence of the strain rate was neglected.

The analysis of the performed tests show that for a right numerical simulation of the contact between a rigid material (not only a rigid material model, but for any material having a stiffness much greater than foam stiffness) with a foam material, some difficulties have to be passed. Between these, the most important issues are how to avoid the error *negative volume* and *complex sound speed* and what to do for a right simulation of the contact.

The answers to these problems are illustrated by the results presented in the table 2 and in the figure 17. The best way seems to be the using of the expanded stress-strain curve. The knowing of the real properties of the foam has a special importance. Unfortunately, experimentally the expanded stress-strain curve can not be obtained or the difficulties are almost impassable.

In this work, we used an experimental stress-strain curve, but only for the strain value of 0.65 [m/m]. The expanded curve was obtained considering an exponential variation of the stress versus strain, by the law:

$$\sigma = \sigma_0 \cdot e^{\alpha(\varepsilon - \varepsilon_0)} \qquad (17)$$

The constant α is determined by the boundary conditions: the values of the σ and ε, at the beginning (σ_o and ε_o) of the expanded curve, which is just the finishing of the experimental stress-strain curve and at the final values taken from technical literature or from experimental curve stress-stretch ration (fig. 2). For the problem presented here, the value of α was 12.813487.

The analysis time plays sometime an important role. As we can see, in the test No_1 and No_2, if the analysis time would have been 0.00285 s or 0.002 s respectively, the numerical simulation could have been considered a right one.

The time variations of the UY nodal displacements, presented in the tests No_3, No_4, No_7 and No_8 are the proof of incorrect contact simulation.

Other parameters which are important in appearing and in over passing the error *negative volume* and *complex sound speed*, as well as for a right working of the contact, are the projectile mass and its velocity.

A final conclusion is that the aspects presented in this work can be very useful for similarly problems referring to the impact with foam materials, but for each case an own research must be made.

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